BCAMT weekly Math Tasks

These open non-curricular math tasks are intended for intermediate to grade 12 students. They are non-curricular so that all students should be able to get started and investigate by drawing pictures, making guesses, or asking questions. When possible, extensions will be provided so that you can keep your students in Flow during the activity. Although they may not fit under a specific topic for your course, the richness of the mathematics comes out when students explain their thinking or show creativity in their solution strategies. I hope you and your class have fun with these weekly mathematics tasks.

June 5, 2016

1. Painted Cube:

Each side of a cube requires paint. You have only two colours of paint, white and red. How many different ways are there to paint the cube? Extensions: What if not two colours? What if not a cube?

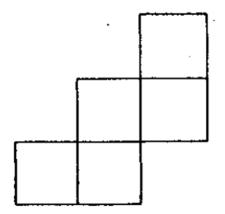
2. People Mover:

There are nine people spaced evenly in a 3x3 grid (try this in your class). The person in the top right corner is removed, leaving an empty space. How can you move the person in the bottom left corner into this empty space? People can only move horizontally or vertically into empty spaces. How many moves are required? Is this the minimum number of moves?

Extensions: What if not 3x3? What if not one empty space?

3. Cube Net:

Can this net be folded into an open-topped box? How do you know?



How many different nets can be folded into an open-topped box? Extensions: What if not open-topped? How many different nets are there for other polyhedra?

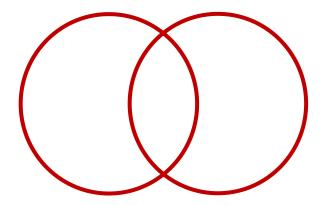
4. **Rock, Paper, Scissors** is considered a fair 2-person game with three options: R, P, S. Why is it fair? Can you design a fair game with 4-options? 5-options? (David Peterson, Fraser Lake Elementary School)

5. Round Peg and Square Hole:

What's a better fit, a round peg in a square hole, or a square peg in a round hole? (http://www.peterliljedahl.com/teachers/good-problem)

6. Jelly Beans:

Place 17 jelly beans into the two circles below so that each circle contains the same number of jelly beans. How many ways can you do this? What if you want the ratio of jelly beans for each circle to be 4:3? m:n?



1. The Tax Collector:

Start with a collection of paychecks, from \$1 to \$12. You can choose any paycheck to keep. Once you choose, the tax collector gets all paychecks remaining that are factors of the number you chose. The tax collector must receive payment after every move. If you have no moves that give the tax collector a paycheck, then the game is over and the tax collector gets all the remaining paychecks.

The goal is to beat the tax collector.

Example:

Turn 1: Take \$8. The tax collector gets \$1, \$2 and \$4.

Turn 2: Take \$12. The tax collector gets \$3 and \$6 (the other factors have already been taken).

Turn 3: Take \$10. The tax collector gets \$5.

You have no more legal moves, so the game is over, and the tax collector gets \$7, \$9 and \$11, the remaining paychecks.

Total Scores:

You: \$8 + \$12 + \$10 = \$30.

Tax Collector: \$1 + \$2 + \$3 + \$4 + \$5 + \$6 + \$7 + \$9 + \$11 = \$48.

From: http://wordplay.blogs.nytimes.com/2015/04/13/finkel-4/? r=0

Extensions: What is the highest score you can achieve? What is the lowest score? What if you had 18 paychecks?

2. Dart Board:

My friend Peter has built a new dart board for his son. The board has two regions: the centre circle, valued at 9 points, and the outside circle, valued at 4 points. What is the largest number that cannot be achieved as a score in this game?

Extensions: What if not 9? What if not 4? What if there was another scoring circle?

3. **The Game of 31.**

Players take turns picking any number from 1 through to 6. Each time a number is picked, it is added to the total score. The player who makes the total score add to 31 wins.

Extensions: What if 31 loses? What if you can choose from 2 through 6?

4. 1001 Loonies:

On a table, there are 1001 loonies lined up in a row. I then come along and replace every second coin with a nickel. After this, I replace every third coin with a dime. Finally, I replace every fourth coin with a quarter. After all this, how much money is on the table?

Extensions: Why is the repeating pattern 12? Design a task that has a repeating pattern of 15. How many starting loonies are needed to make a total of \$100?

5. Die Hard:

You have a 3 L jug and a 5 L jug and an unlimited supply of water. How can you measure exactly 4 L of water? https://youtu.be/BVtQNK ZUJg

Extensions: What if not 4 L? What if not 3 L and 5 L? What if the jugs were 3 L and 6 L? or 3 L and 7 L?

6. **Flip a card – Toss a card** (from Peter Liljedahl). https://youtu.be/FOcqqV0IdQ8

Start with the cards ace, two, three, four, and five. Arrange the cards in such a way so that they come out in increasing sequence when you deal the cards out like this:

- i) top card place on table
- ii) next card place at bottom of deck
- iii) repeat this process until all cards are on the table.

What is the pattern? What is your strategy?

Extensions: Add more cards to the deck. Can you do 13 cards? 52 cards? 104 cards?

7. Four 4's:

Write each number from 1 - 10 using exactly 4 fours and any mathematical sign or symbol.

Extensions: 1 – 20 or 1 – 100. Can you do this with 4 fives or 4 threes etc...

8. The frog puzzle.

Three green frogs are trying to change position with 3 orange frogs. Green frogs and orange frogs can only move forward onto an empty lily pad or leap frog a single frog onto an empty lily pad.



http://britton.disted.camosun.bc.ca/frog_puzzle.htm

How many moves are required to solve this puzzle?

Extension: What if there were 4 frogs on each side (or 5 frogs or *n* frogs...)? What if the number of frogs on each side is not equal? How can you communicate the solution to a friend over the phone?

Primary Math Task

A. How Many are Hiding?

(https://www.youcubed.org/task/how-many-are-hiding/)

Material:

- 10 or more snap cubes /objects per player
- a cup for each player

Task Instruction

- In this activity each child has the same number of cubes and a cup.
- They take turns hiding some of their cubes in the cup and showing the leftovers.
- Other children work out the answer to the question "How many are hiding," and say the full number combination.

Example: I have 10 cubes and I decide to hide 4 in my cup. My group can see that I only have 6 cubes. Students should be able to say that I'm hiding 4 cubes and that 6 and 4 make 10.

October 2, 2016

9. Rope around the Earth:

A rope is wrapped tight around the Earth along the equator. The rope is cut, and 1 m of rope is added to the length and then stitched back together. A big green super hero lifts the rope and throws it so hard that it enters into circular orbit the Earth (bonus for the physicists: how fast does the rope need to spin to be in orbit?). How high is the rope hovering over planet Earth? (Earth radius is 6371 km)

Extensions: What about Jupiter, the Sun, or a basketball? Instead of spinning the rope, the super hero lifts the rope straight up until it is tight again. How high can she lift the rope?

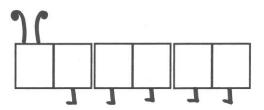
10. Lockers:

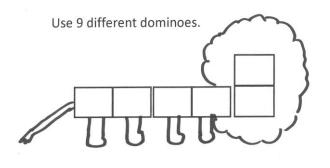
Imagine that you are at one end of a hallway with 1000 open lockers. The first student goes along and closes every single locker. A second student then goes and opens every second locker. A third student then changes the state of every third locker (meaning if the locker is open, the student closes it, and if the locker is closed, the student opens it). A fourth student changes the state of every fourth locker. This continues until 1000 students have gone through. When finished, which lockers are closed?

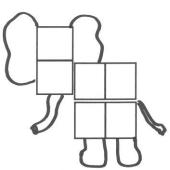
Extensions: Why are these lockers closed? What if they are not lockers, but they are dials with three positions: A, B or C?

B. Dominoes: K – Grade 1

Use a set of 'double six' dominoes. Make each animal using 10 dots. Use 9 different dominoes.







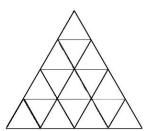
Encourage students to make their own animals using univerent quantities (i.e. make animals using 15 dots in total.)

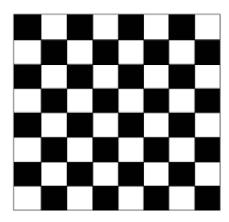
October 9, 2016

11. Chess Board:

How many squares are there on a chess board? And no, the answer is not 64.

Extensions: How many rectangles? How many triangles in this?





12. Toaster:

Three slices of bread are to be toasted under a grill. The grill can hold two slices at once but only one side is toasted at a time. It takes 30 seconds to toast one side of a piece of bread, 5 seconds to put a piece in or take a piece out and 3 seconds to turn a piece over. What is the shortest time in which the three slices can be toasted?

(Thinking Mathematically by John Mason)

Extensions: The answer is not 151 sec. What about 4 slices? 5 slices?

C. Snap:

Material: 10 or more snap cubes per student

Task Instruction

- This is an activity that children can work on in groups.
- Each child makes a train of connecting cubes of a specified number.
- On the signal "Snap," children break their trains into two parts and hold one hand behind their back.
- Children take turns going around the circle showing their remaining cubes.
- The other children work out the full number combination.

(https://www.youcubed.org/task/snap-it/)

October 16, 2016

13. Fibonacciish:

Consider the following pattern of 5 whole numbers, where each number is the sum of the previous two numbers:

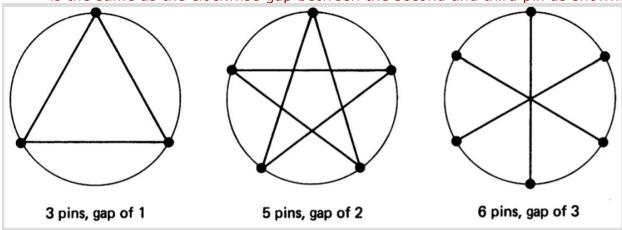
I want the 5th number to be 100.

Find all the whole seed numbers that will make this so (3 and 12 are the seed numbers in the above sequence). (Peter Liljedahl)

Extensions: How do you know you have them all? How about integers?

14. Threads and Gaps:

A number of pins are placed around a circle. A thread is tied to one pin, and then looped tightly around a second pin. The thread is then looped tightly round a third pin so that the clockwise gap between the first and second pin is the same as the clockwise gap between the second and third pin as shown:



The process is continued, always preserving the same clockwise gap until the first pin is reached. If some pin has not yet been used, the process starts again. Five pins with a gap of two use just one thread, while six pins with a gap of three use three threads. How many pieces of thread will be needed in general?

(Thinking Mathematically, John Mason)

Extensions: What if the gap alternates between two numbers?

Primary Math Task

D. Making 5:

Take your class outside and have students collect 5 of an object (leaves, rocks, etc..). The task is for students to work in groups and find different ways to make 5. How many ways can you make 5? How can you show all of your ways?

Extensions: How about 4? How about 6?

October 23, 2016

15. Cartesian Chase (from <u>Thinking Mathematically</u> – John Mason)

This is a game for two players on a rectangular grid with a fixed number of rows and columns. Play begins in the bottom-left-hand square where the first player puts his mark. On his turn a player may put his mark into a square directly above or directly to the right of or diagonally above and to the right of the last mark made by his opponent. Play continues in this fashion, and the winner is the player who gets their mark in the upper-right-hand corner first.

Find a way of winning which your great aunt Maud could understand and use.

Extensions: What if you cannot move diagonally? What if the top right square means that the player loses?

16. Silver Coins:

You have 10 silver coins in your pocket (silver means that the coins could be any of nickels, dimes or quarters). How many different amounts of money could you have?

Primary Math Task

E. Attributes of Shapes:

Allow students time to explore the attributes of various 3-D shapes. Have them identify the faces, edges and vertices of the 3-D shapes? Present various problems for them to solve:

- 1. If you had 3 cones, 2 cylinders and a sphere, how many faces would you have? How do you know?
- 2. You have 1 cube and your friend has 4 cylinders. Who has more faces? How do you know?
- 3. I have some objects and in total I counted 8 faces. What might the objects be? Explain your thinking.
- 4. I have a collection of objects that have 7 faces and a point. What shapes could they be? Explain your thinking.

Have the students create their own clues to create a problem.

(From Vector, March 2015)

October 30, 2016

17. Pen Caps:

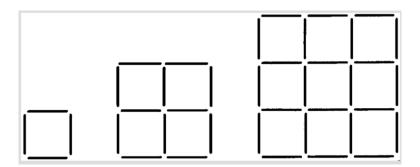
You have 7 pen caps on a table with the same side up. These caps all have to be turned over, but you can only turn over exactly three at a time. What is the smallest number of moves you can do this in? How do you know it is the smallest?

(https://www.youcubed.org/task/seven-flipped/)

Extensions: What about 8, 9, or n pen caps? What if you turn over 4 at a time? What if the pen caps had a middle position: up, middle, down?

18. Match Sticks:

How many matches are required to make n² unit squares in a square array as in the following sequence? (<u>Thinking Mathematically</u> – John Mason)



Extensions: What about a triangular array? What about a 3D cubical array?

F. Number Patterns:

I used digit cards to create a 2-digit number pattern. The wind blew the cards and mixed them up. How might you place the loose digit cards into the following to complete a pattern? How do you know? How might you extend the pattern? (Vector, December 2015)

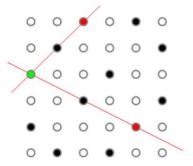




November 6, 2016

19. **Just Two Dots**

On a 6 by 6 square array, dots can be placed at any intersection. If a line is drawn between any two dots, then this line can only pass through exactly two dots. Can 12 dots be placed on this array? This picture shows a mistake, because the green dot is a third dot on two lines.



(adapted from: http://galileo.org/classroom-examples/math/math-fair-problems/puzzles/minaret-with-a-view/)

Extensions: What about a 5x5 or 7x7? What if each line must contain exactly three dots?

20. **Dragon Fractal**

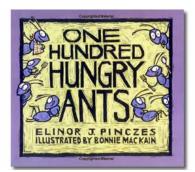
Imagine a long strip of paper folded in the same direction once, twice, and then a third time. When the strip is unfolded, how many creases will be on the paper? What directions will the creases be pointing? What about n folds?

Extensions: When the paper is unfolded, and the creases made to equal 90°, what do you notice in the shapes?

G. Hungry Ants:

100 Hungry Ants by Elinor J. Pinczes Read the story to the students.

Ask the students to choose one of the following numbers: 12, 24 or 36.



Ask them to imagine that this number of ants is going to the picnic.

Ask: How many different ways could the ants arrange themselves into equal rows?

Have the students draw an array and write an equation for each solution.

21. Cottages

A circular road is 27 km long. On this road are six cottages, owned by 6 friends. The friends visit each other a lot, and they have noticed that every whole number from 1 to 26 (inclusive) is accounted for at least once when they calculate the distances from one cottage to another. Of course the friends can walk in either direction as required. Your task is to place these cottages at distances that will fulfill these conditions. (Source: 536 Puzzles and Curious Problems by Henry Ernest Dudeney)

Extensions: Can you find more than one solution?

22. **30 Link Chain**

You are backpacking through Europe. You would like to stay in the South of France for as long as possible, but you have run out of money. However, you have a 30 link gold chain and you have found a hotel that is willing to accept one link per night for payment of room and board. The manager wants payment every day and he is willing to help you out by cutting links for you. The problem is that he wants one gold link payment for every link he cuts. What is the most number of nights that you can stay in the hotel? (Source: http://www.peterliljedahl.com/teachers/good-problem)

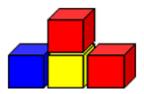
Extensions: What if the manager will accept payment after every second day?

H. Up-and-Down Staircase:

One block is needed to make an up-and-down staircase, with one step up and one step down.



4 blocks make an up-and-down staircase with 2 steps up and 2 steps down.



How many blocks would be needed to build an up-and-down staircase with 5 steps up and 5 steps down?

Explain how you would work out the number of blocks needed to build a staircase with any number of steps.

(from: https://nrich.maths.org/2283/note)

23. **Counterintuitive tasks**. What is it that makes them surprising?

- (1) A bat and a ball cost \$1.10 in total. The bat costs \$1.00 more than the ball. How much does the ball cost?
- (2) If it takes 5 machines 5 minutes to make 5 widgets, how long would it take 100 machines to make 100 widgets?
- (3) In a lake, there is a patch of lily pads. Every day, the patch doubles in size. If it takes 48 days for the patch to cover the entire lake, how long would it take for the patch to cover half of the lake?

(From: The Journal of Economic Perspectives, Vol. 19, No. 4 (Autumn, 2005), pp. 25-42)

24. **Factor Craze**

Prime numbers have exactly two factors – 1 and itself. Which numbers have exactly 3 factors? Exactly 4 factors? And so on. Given any positive integer, n, how can you tell exactly how many factors it has?

I. Creature's Legs:

At the park, Mike counted (6, 10, 14 – Choose a quantity appropriate for your students) creature's legs.

What creatures could there have been at the park?

Which combination of creatures show the number of legs counted?

Show more than one combination.

November 27, 2016

25. **N-gons**

How many diagonals are there in an n-gon (a polygon with n sides)?

Extensions: What about concave n-gons? What about n-hedrons?

26. **Approaching Midnight**

It is 6:00. With a partner, take turns adding one of 15 min, 30 min, 45 min, or 60 min to the clock. The first player to reach 12:00 wins. (adapted from: http://wild.maths.org/approaching-midnight)

Extensions: Is there a winning strategy? What if the first to 12:00 is the loser?

J. Watermelon Seeds:

One hot day, my dad cut a slice of watermelon for me to eat. I counted 13 (change number to meet the needs of the students - 23 or 33) black and white seeds in the slice. There were more black than white seeds. How many of each kind of seed might there be?

27. Handshakes

There is a party for mothers and daughters. All of the mothers shake hands among themselves. Every daughter shakes hands with only the mothers. How many handshakes are there if the party involved 17 mother-daughter pairs?

Extensions: What if the mothers do not shake hands with their own daughters? What if grandmothers were brought into the mix?

28. Milkcrate

A certain milkcrate can hold 36 bottles of milk. Can you arrange 14 bottles in the crate so that each row and column has an even number of bottles? (<a href="https://doi.org/10.1001/jhinking.new1.5001/jh

Extensions: What is the smallest array that can fit 14 bottles under this rule? What about 15 bottles?

K. Coloured Dice

Roll 3 different coloured dice. What are all the possible ways to get a total of 5 points?

29. **Tethered Goat**

A goat is tethered by a 6 metre rope to the outside corner of a shed measuring 4 m by 5 m in a grassy field. What area of grass can the goat graze? (Thinking Mathematically – John Mason)

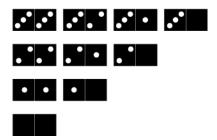
Extensions: What if the rope was fastened to the middle of one wall? What if the rope was 20 m long? What if the shed was circular?

30. **Mountain Bike Race**

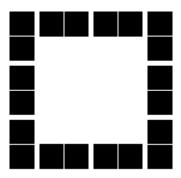
Mountain bike Race. There are 25 racers and the track can only fit five racers at any time. Devise a strategy to determine gold, silver, and bronze. How many races are necessary?

L. Dominoes

These are the 'double-3 down' dominoes.



Use these dominoes to make this square so that each side has eight dots.



(http://nrich.maths.org/146)

31. Fifteen

Nine counters marked with the digits 1 to 9 are placed on the table. Two players alternately take one counter from the table. The winner is the first player to obtain, amongst his or her counters, three with the sum of exactly 15. (Thinking Mathematically – John Mason)

Extensions: What are some strategies for winning? What childhood game does this connect with?

32. **Mixing Sodas**

Over the holiday season, I found two crazy soda flavours: Peppermint and Prune.

I poured a tall glass of each and then decided to mix them. I carefully measured 30 mL of the Peppermint soda and poured it into the Prune soda glass. After stirring the Prune soda mix, I measured 30 mL of this mix and poured it back into the Peppermint soda glass, creating a Peppermint soda mix. After all of this, which glass is purer? In other words, is the Peppermint mix more pepperminty or is the Prune mix more Pruney?

Extensions: What if we introduce a third soda: Cabbage flavour?

M. Trade Ya!

- 1. Use pattern blocks for this task.
- 2. Explain that each of the pattern blocks has a different value.

(note: the prices are proportional to the areas)

- a. Yellow hexagons are worth \$6
- b. Red trapezoids are worth \$3
- c. Blue rhombi are worth \$2
- d. Green triangles are worth \$1
- 3. Challenge the students to find as many designs as possible with the value of \$20. What relationships do they notice? How did they trade pattern blocks to create other possibilities? (<u>from Vector, Fall 2016</u>)

January 8, 2017

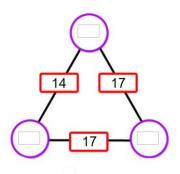
33. **Square Cake**

A square cake is to be divided amongst 5 people so that each person has equal portions of cake and icing. How should you cut the cake?

Extensions: What about a triangle cake? Hexagon? What about 3 people?

34. **Arithmagons**

A secret number is assigned to each vertex of a triangle and pairs of numbers are added together to equal the number given along each edge. The task is not to solve each Arithmagon (although this is a good first step); rather, the task is to find structure, relationships and strategies for solving all Arithmagons.



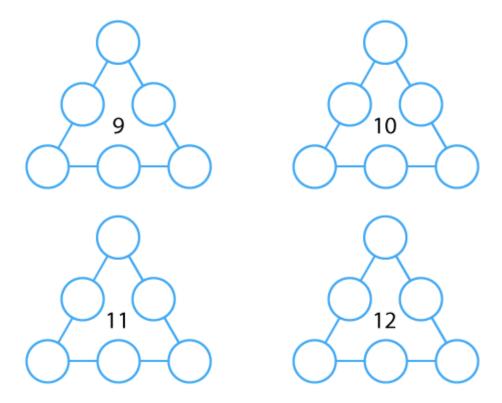
Extensions: Investigate other polygons.

(From NRICH and Thinking Mathematically: https://nrich.maths.org/2670)

N. Triangles

Arrange the numbers 1 to 6 in each set of circles below.

The sum of each side of the triangle should equal the number in the centre of the triangular shape.



Extension: Are there other centre numbers that work? Can you do this with squares?

(From NRICH: https://nrich.maths.org/188)

35. Palindromes

Consider a two-digit number: for example, 84. 84 is not a palindrome, so reverse the digits and add it to the original number: 84 + 48 = 132. This is still not a palindrome, so try it again: 132 + 231 = 363. 363 is a palindrome, so 84 can be called a depth 2 palindrome. Find the depth of all two-digit numbers.

Extensions: What about 3-digit numbers? What about the depth for the second time of becoming a palindrome? What happens when you shade a Hundreds Chart according to the number's depth?

(http://www.magic-squares.net/palindromes.htm)

36. **6-sided die**

A regular 6-sided die has opposite sides that add to seven. If you removed this restriction, how many different 6-sided die can you make?

Extensions: What about 3-sided and 8-sided die? What if a 6-sided die has 2 of the same numbers (1, 1, 2, 3, 4, 5) or 3 of the same numbers (1, 1, 1, 2, 3, 4)?

O. Game of War

This game requires a deck of playing cards where the face cards have been removed.

The Deal

The deck is divided evenly, with each player receiving 20 cards, dealt one at a time, face down. Anyone may deal first. Each player places their stack of cards face down, in front of them.

The Play

Each player turns up a card at the same time and the player with the higher card takes both cards and puts them, face down, on the bottom of their stack.

If the cards are the same rank, it is War. Each player turns up one card face down and one card face up. The player with the higher cards takes both piles (six cards). If the turned-up cards are again the same rank, each player places another card face down and turns another card face up. The player with the higher card takes all 10 cards, and so on.

How to Keep Score

The game ends when one player has won all the cards. -

(http://www.bicyclecards.com/how-to-play/war/#sthash.hbicGSDQ.dpuf)

37. Painted Cube

Paint all the sides of a $3 \times 3 \times 3$ cube. Once it is dry take it apart into its $1 \times 1 \times 1$ unit cubes. How many of these unit cubes have paint on three faces? Two faces? One face? No faces? Explore for $4 \times 4 \times 4$, $n \times n \times n = \frac{http://www.peterliljedahl.com/teachers/good-problem)}{http://www.peterliljedahl.com/teachers/good-problem)}$

Extensions: What about n x n x m? or n x m x p?

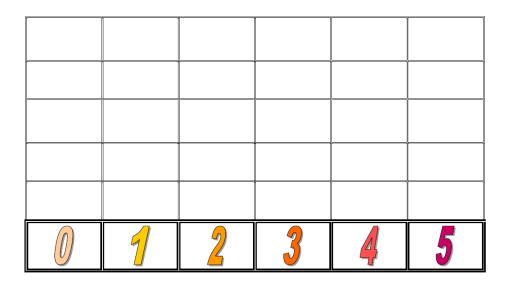
38. **Triangular Numbers**

The numbers 1, 3, 6, 10, 15, ... are known as triangular numbers because each number can be made into a triangular array of dots. What patterns do you notice in these numbers? What is the largest triangular number less than 500? Less than 5000?

Extensions: Can you find the nth triangular number? What numbers could be considered pentagonal or hexagonal?

P. Subtraction Graph

2 play this game. Use 2 dice and markers. Roll the dice and subtract the lower number from the higher. Put a marker in the first empty square above your answer. A player wins when they place a marker that is the first to reach the top of the graph.



(From the Surrey School District)

39. **Pirates**

A band of 10 pirates are going to disband. They have divided up all of their gold, but there remains one GIANT diamond that cannot be divided. To decide who gets it the captain puts all of the pirates (including himself) in a circle. Then he points at one person to begin. This person steps out of the circle, takes his gold, and leaves. The person on his left stays in the circle, but the next person steps out. This continues with every second pirate leaving until there is only one left. Who should the captain point at if he wants to make sure he gets to keep the diamond for himself? What if there were 11 pirates? What if there were 12 pirates? What if there were 27 pirates? Etc.

(From: http://www.peterliljedahl.com/teachers/good-problem)

Extensions: What if two pirates are skipped over? What if The Captain wants himself and his first lieutenant to be the two winners?

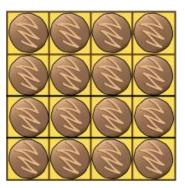
40. **Marching Band**

Students in a marching band want to line up for their performance. The problem is that when they line up in 2's there is 1 left over. When they line up in 3's there are 2 left over. When they line up in 4's there are 3 left over. When they line up in 5's there are 4 left over. When they line up in 6's there are 5 left over. When they line up in 7's there are no students left over. How many students are there? (From: John Grant McLoughlin)

Extensions: What if there are over 200 students in the band? What if there are 6 left over when lined up in 7's?

Q. Box of Chocolates

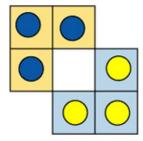
Erica doesn't like odd numbers, so the box of chocolates shown to the left meets with her approval. The problem is that she has to remove six chocolates from the box in such a way that she leaves an even number of chocolates in each row and each column.



(From: Boris Kordemsky's Moscow Puzzles.)

41. **Switch Positions**

The picture shows a 3 by 3 board with two missing corners. There are 3 blue chips on the yellow part of the board and 3 yellow chips on the blue part of the board.

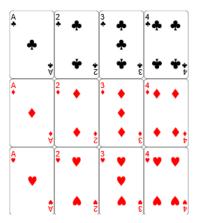


The task is to put the blue chips on the blue part and the yellow chips on the yellow part. The chips can only move horizontally or vertically into an empty space or they may leapfrog a single chip into an empty space. (Adapted from: http://www.mathfair.com/other-puzzles.html)

Extensions: What is the minimum number of moves? How do you know it is the minimum number of moves? What if you expand the board?

42. **Game of 22**

Arrange three rows of cards from ace to 4 as shown below.



Two players alternately choose a card and add it to the common total. The winner is the player who makes 22 or who forces the other player to go beyond 22. What is a winning strategy? (Adapted from: http://www.mathfair.com/other-puzzles.html)

R. 5 Cubes

Using exactly 5 interlocking cubes, make as many shapes as you can so that all five cubes are touching the table. How many *different* shapes can you make? (Vector: Spring 2011)



43. **Square Peg – Round Hole**

What fits better: A square peg in a round hole or a round peg in a square hole? (From: http://www.peterliljedahl.com/teachers/good-problem)

44. Multi-facets

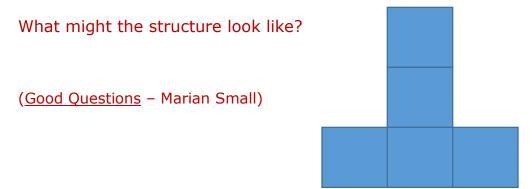
Picture to yourself a length of rope, lying on a table in front of you. The cross section of the rope is a regular N sided polygon. Slide the ends of the rope towards you so that it almost forms a circle. Now, mentally grasp the ends of the rope in your hands. You are going to glue the ends of the rope together but before you do, twist your right wrist so that the polygonal end rotates through one nth of a full revolution. Repeat the twisting a total of T times, so that your mental wrist has rotated through T nths of a full revolution. NOW, glue the ends together, so that the polygonal ends match with edge glued to edges.

When the mental glue has dried, start painting one facet (flat surface) of the rope and keep going until you find yourself painting over an already painted part. Begin again on another facet not yet painted, and use a different colour.

How many colours do you need? (Thinking Mathematically – John Mason)

S. What is the shape?

A shape is made with linking cubes. When you look at it from one side, it looks like this:



45. **3D Tic-Tac-Toe**

A marble can be placed in each of the 27 cubes that make a larger 3x3x3 cube. Imagine a 3D tic-tac-toe game where players take turns placing their own marbles in any of the 27 cubes. A player wins when they have 3 marbles in any straight line. How many different ways can someone win in this game?

(Adapted from http://nrich.maths.org/895)

Extensions: Investigate 4x4x4, 5x5x5, and nxnxn. What about a hypercube?

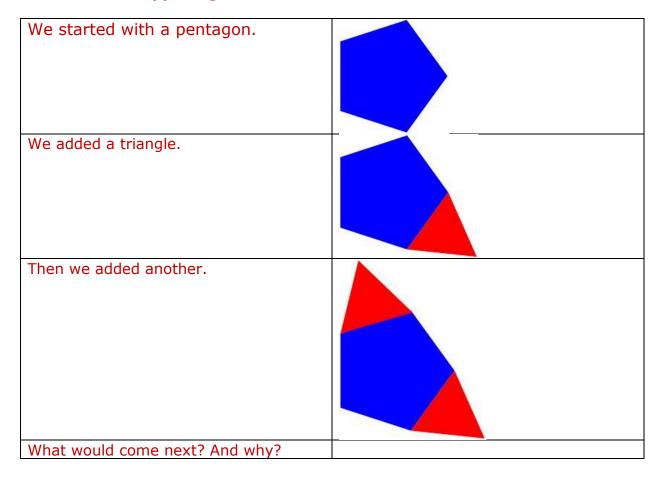
46. **Sharing Bacon**

You are a chef at a summer camp and you are frying 30 identical strips of bacon for this morning's breakfast. A counselor comes in to inform you that there are only 18 campers coming in for breakfast and they all love bacon. What is the minimum number of cuts necessary? What is the minimum number of pieces?

(Adapted from Thinking Mathematically – John Mason)

Extensions: How do you know it is the minimum? What about sharing amongst 17 campers? 16 campers? n campers?

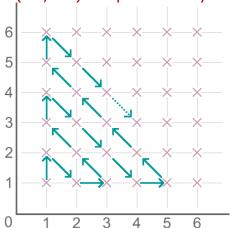
T. What's Happening



(http://nrich.maths.org/7810)

47. **Route to Infinity**

If the pattern of arrows continues for ever, which point will the route visit immediately after (18, 17)? Explain how you know.



(http://nrich.maths.org/5469)

Extensions: How many points will be visited before the route reaches the point (9, 4)? Which point will be the 1000th to be visited.

48. The Last Number

Consider the string 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. Cross out any two numbers in this list and add the difference to the end of the list. This new number is now part of the list. Continue the process of crossing out two number on the list and adding the difference until there remains only one number. What can you say about the last number? Explore. (From: Richard Hoshino)

U. Numbers and Words

Create a sentence that uses each of the four numbers and words shown below. Other numbers and words can also be used.

3, more, 5, and

49. **Magic Squares**

A magic square is a square grid with n rows and n columns, filled with distinct numbers from 1 to n^2 , such that the sum of the numbers in each row, column, and both long diagonals is the same.

- 1. Can you come up with a 2×2 magic square?
- 2. What about a 3×3 magic square?
- 3. What value does each row, column, and long diagonal need to sum to in a nxn magic square?

(From: https://tinyurl.com/jrwxt7l)

Extensions: Investigate magic rectangles and magic triangles.

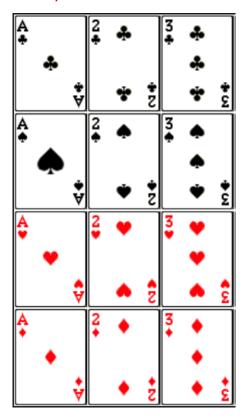
Students will enjoy this video: https://youtu.be/50EXodrHGWU

50. **Box of Marbles**

In a box, you have 13 white marbles and 15 black marbles. You also have 28 black marbles outside the box. Remove two marbles, randomly, from the box. If they are of different colours, put the white one back in the box. If they are of the same colour, take them out and put a black marble back in the box. Continue this until only one marble remains in the box. What colour is the last marble? (From Vector 55(1) – p. 49: https://tinyurl.com/zaksq6b)

V. Making Numbers with Cards

Each group of students has a set of playing cards Ace to three (Ace has a value of 1). Students are to use these 12 cards to make the numbers from 1-10 in a variety of ways.



51. **Bridge Crossing**

Five persons are standing on one side of a bridge.

They want to cross the bridge.

Without a torch, they cannot proceed.

Only one torch is available.

The torch has a remaining battery life of only 30 seconds.

Only two people can go over the bridge at one time.

The torch needs to be returned to the remaining persons.

The five people take different times to cross the bridge.

One takes 1 second to cross the bridge.

The others take 3 seconds, 6 seconds, 8 seconds and 12 seconds.

Everyone crosses the bridge within 30 seconds.

How do they proceed?

Extensions: What if the time taken for two to cross is the average of their two times?

52. **Pumpkins**

A 600-pound pumpkin was entered in a contest. When it arrived, it was 99% water. The pumpkin sat for days in the hot sun, lost some weight (water only), and is now 98.5% water. How much does it now weigh?

Extensions: At 98.5%, the pumpkin had lost 0.5% water. What if the pumpkin loses 1%, 2%, n%?

(From Vector 51(1) - p. 41: https://tinyurl.com/k3q6rsy)

W. Sharing Cookies

Charlie, Susan, and Amber get to share six cookies. However, Susan's mother has told her that she is only allowed to have one cookie. How do you share the cookies?

(From Vector 51(1) - p. 41: https://tinyurl.com/k3q6rsy)

53. **25 coins**

25 coins are arranged in a 5 by 5 array. A fly lands on one, and tries to hop on to every coin exactly once, at each stage moving only to the adjacent coin in the same row or column. Is this possible?

Extensions: Can you explain why some starting locations are not possible? What about 3D? What about rectangles?

(<u>Thinking Mathematically</u> – John Mason)

54. **Pancakes**

When I make pancakes, they all come out different sizes. I pile them up on a plate in the warming oven as they are cooked, but to serve them attractively, I would like to arrange them in order with the smallest on top. The only sensible move is to flip over the topmost ones. Can I repeat this sort of move and get them all in order?

(<u>Thinking Mathematically</u> – John Mason)

Extensions: What is the most complicated arrangement for 3, 4 or 5 pancakes? What is the minimum number of flips for these arrangements?

(this is an open mathematics problem. See: https://en.wikipedia.org/wiki/Pancake_sorting)

X. Names and Numbers

Take a photo of each student and mount on a cardstock card with their name written on it. Have each student find a point value of 1-26 for their first name by beginning at A=1, B=2, and continuing for each letter of the alphabet. How did you figure out the value of your name? What strategies did you use? (Make sure you spend the time debriefing the various strategies with the whole group). What do you notice about your names compared to others' names? Have students compare the value of the first names (last names, full name, family member names...).

A = 1	I = 9	Q = 17	Y = 25
B=2	J = 10	R = 18	Z = 26
C = 3	K = 11	S = 19	
D = 4	L = 12	T = 20	
E = 5	M = 13	U = 21	
F = 6	N = 14	V = 22	
G = 7	O = 15	W = 23	
H = 8	P = 16	X = 24	

(From Vector 55(3) - p. 61: https://tinyurl.com/kog7oyj)

55. Hotel Rooms

There is a hotel with ____ rooms in a row. Every room has a door to the corridor and doors connecting it to the adjacent rooms. A woman has rented all ____ rooms, telling the hotel manager that if he needs to contact her, she will always be in a room adjacent to the room she was in the day before. The hotel manager needs to contact the woman regarding an issue with her credit card; however, he can only check one room each day.

Can you devise a scheme such that the hotel manager is guaranteed to find the woman?

Start this task with 3 rooms, and when groups have convincing well-reasoned solutions, move to 4 rooms, 5 rooms, and so on.

56. Aliens!

Aliens from Jupiter capture three men. The aliens give the men a single chance to escape uneaten.

The captives are lined up in order of height, and are tied to stakes. The man in the rear can see the backs of his two friends, the man in the middle can see the back of the man in front, and the man in front cannot see anyone. The aliens show the men five hats. Three of the hats are black and two of the hats are white.

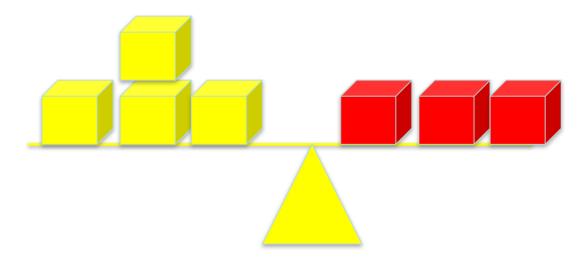
Blindfolds are then placed over each man's eyes and a hat is placed on each man's head. The two hats left over are hidden. The blindfolds are then removed and it is said to the men that if one of them can guess what color hat he is wearing they can all leave unharmed.

The man in the rear who can see both of his friends' hats but not his own says, "I don't know". The middle man who can see the hat of the man in front, but not his own says, "I don't know". The front man who cannot see ANYBODY'S hat says "I know!"

How did he know the color of his hat and what color was it?

Y. How Many?

How many might be in each yellow box? How many in each red?



(From Marian Small: https://tinyurl.com/ll7yvm8)

57. Chicken Nuggets

Chicken nuggets come in boxes of 6, 9 and 20.



What is the largest number of nuggets that you cannot buy when combining various boxes?

(From: http://www.playwithyourmath.com/)

58. **Split 25**

Take the number 25, and break it up into as many pieces as you want.

$$25 = 10 + 10 + 5$$

 $25 = 2 + 23$
 $25 = 1 + 1 + 1 + 1 + 1 + 1 + 1 + 9 + 9$

What is the biggest product you can make if you multiply those pieces together?

Will your strategy work for any number?

(From: http://www.playwithyourmath.com/)

Z. Number Path

You are on a number path made up of squares of numbers starting at 1 and continuing as far as you wish...



You move SOME steps forward. Then you move SOME steps back. You repeat both moves. You land at 9. How many steps each way?

(From Marian Small: https://tinyurl.com/ll7yvm8)

59. **Consecutive Sums**

Which positive integers can be written as the sum of two or more consecutive positive integers? What do you notice? What do you wonder?

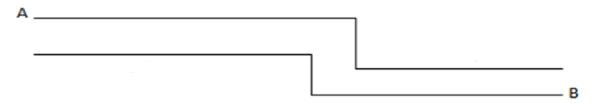
Extensions: In how many ways can a number, n, be written as the sum of two or more positive integers?

(From: http://www.playwithyourmath.com/)

60. The Lab Rat

A lab rat released into a maze has been noticed to only run in exactly straight lines and to always traverse the maze in the shortest possible distance. Given the maze section below (not drawn to scale):

a) What will be the path for the rat to travel from A to B?



b) After the measurements are provided, what will be the shortest distance the rat covers from A to B?



(Adapted from Vector 2014: http://bit.ly/2pV8ldm)

A1. Moving Colours

Give each student a colored circle (or use different shapes) red or yellow (or other colors) that you have pre-prepared. We don't recommend using gender or other personal characteristics. There should be equal numbers or one more of one of the colors.

Ask students, "How many students have red circles and how many have yellow circles." Encourage them to get up and move around the room to work this out.

Ask students, "How can we show that we have an equal number of each color or more of one color than the other color?"

During these questions observe students interactions as they try to problem solve and justify.

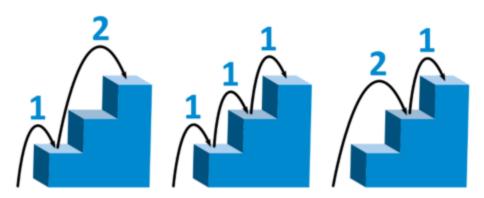
Ask students, "How many students can fit in a row on the carpet?" "How many rows will we have?" "What would be the best arrangement?"

As students complete each task, record the number sentence on the board. After students have thought about the carpet area-seating question and have provided a solution you can record names and have your carpet seating chart complete.

(From: https://www.youcubed.org/task/moving-colors/)

61. Climbing Steps

If you can only climb one step or two steps at a time, in how many ways can you climb 3 steps? 4 Steps? 10 steps? 15 steps? *n* steps?



Extensions: Why is it that Fibonacci shows itself in this task? What if you can also climb three steps at a time?

(From: http://www.playwithyourmath.com/)

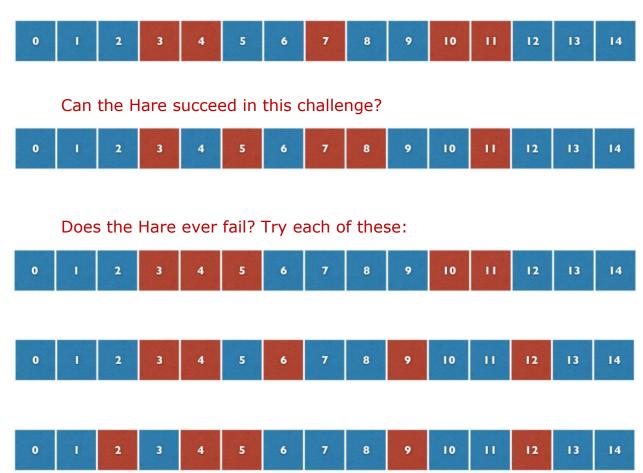
62. Handshake Routine

A large group has agreed that at one-minute intervals, they will each participate in a "handshake" with one other person. During each of these handshake routines, no person is left out and no handshake is repeated. The handshake routine continues until each person has shaken every other person's hand exactly once. This routine is possible in some groups and impossible in others. In groups where it is possible, show how it can be done.

(Adapted from: http://mathpickle.com/project/ruffian-ritual/)

B1. The Tortoise and the Hare

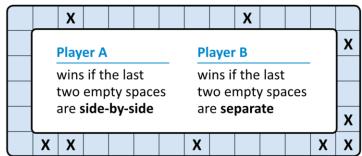
The Tortoise has challenged the Hare to a hopping competition. The challenge is for the Hare to complete 3 equal hops and not land on a red square. Can the Hare succeed in this challenge?



(Adapted from: http://mathpickle.com/project/tortoise-and-hare-the-revenge-race-skip-counting-pattern/)

63. **Space Race**

In the game of Space Race, two players take turns writing an **X** in an empty space.



What is the best strategy for Player A? What is the best strategy for Player B? Who should go first?

Extensions: What about s smaller board? What about an odd number of squares? (From: http://www.playwithyourmath.com/)

64. Tower of Hanoi



The object of this puzzle is to move the tower of rings (students can use linking cubes) from one peg to another following these two rules:

You can only move one ring at a time.

You can never place a ring on top of a smaller ring.

The optimal solution for a 3-ring puzzle requires 7 steps. For this task, students can investigate the relationship between the number of rings and the optimal number of steps.

Extensions: What is the reason behind the patterns that you notice? What about 4 pegs? Interestingly, the optimal solution for the 4-peg variation is an open math problem.

C1. Play with 60

You have 60 items in a bowl; how could you arrange them to make it easier for a friend to count? Can you organize them in different ways? You want to put the 60 items into bowls so that there is the same number in each bowl; how many different bowls will you need so that there are no items left over? (From Vector Spring 2017)

65. Rollover Numbers

To make a rollover number:

1. Pick a four-digit number.

2. Transfer the first digit to the end.

3. Add the new number to the old number.

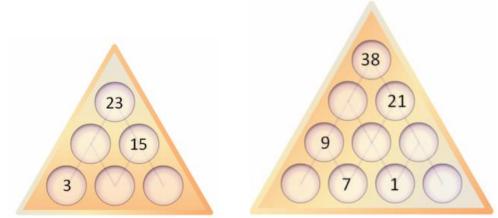
3575

Which of these numbers are rollover numbers? 9867 8612 13859 4322

Extensions: How do you know when a number is not rollover? (From: http://www.playwithyourmath.com/)

66. **Triangle Patterns**

To complete these puzzles, the sum of two horizontally adjacent cells equals the cell above.



What is the minimum number of clues necessary for any puzzle to have a unique solution? Does it matter where these clues are located?

Extensions: What about a 3-D version of this puzzle?

(From: https://nrich.maths.org/2281)

D1. Add and Subtract

I add two numbers. I also subtract them.

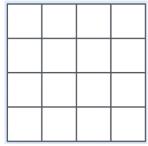
The "add answer" is 10 more than the "subtract answer."

What could the numbers be?

(From: Marian Small NCTM annual 2017)

67. **Truth-Tellers**

Truth-tellers always tell the truth and are marked with an 'X' on the grid. Liars always lie and are marked with an 'O.' When asked the question, "Are you next to exactly two like yourself?" Everyone responded, "YES!" Where are the truth-tellers and the liars?



Extensions: Is this a unique solution? How do you know? What about

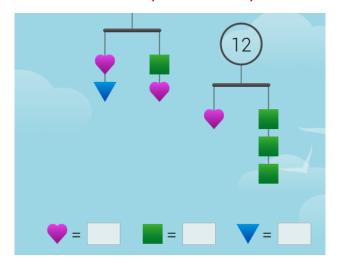
a 5x5 grid, 6x6,...?

(Adapted from: http://mathpickle.com/project/pinoccchios-

playmates/)

68. **SolveMe Mobiles**

Determine the value for each shape. How do you know?



Extensions: Can you build your own SolveMe Mobile?

(From: https://solveme.edc.org/Mobiles.html)

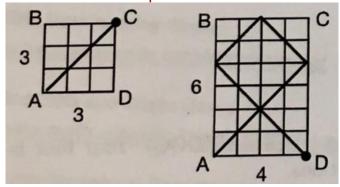
E1. Sorting Numbers

You sort whole numbers. 4 and 19 go in one category, but 3 and 21 go in a different category. What might the categories be?

(From: http://www.onetwoinfinity.ca/good-question/)

69. **Billiards**

Hit a ball at a 45° angle from the lower left corner, A, of a rectangular billiards table. The ball rebounds off each side in a new direction, but at the same angle. As the dimensions of the billiards table varies, explore which of the four corner pockets the ball can end up in.



Extensions: How many bounces? What if there are extra pockets at the sides?

(Adapted from a workshop at Changing the Culture 2017 by Jamie Mulholland and Richard Hoshino)

70. **Zombies**

You are trying to survive a zombie apocalypse by running into every square exactly once. The numbers on every square represent the number of zombies in that area. When running, you are only able to move into adjacent squares (not diagonally). When you need to decide on a path to take, you will always choose the square with the least number of zombies.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Which starting squares will result in survival?

Extensions: What about larger squares? What happens if you run into squares with more zombies?

F1. TOUCH counts

Download the free app onto iPads for students.

Have students explore and play within the Numbers World



Try these tasks:

- Count to 10, either above or below the shelf.
- Tap two fingers at the same time. Keep doing that until you get to 20.
- Teacher places 1, 2 and 5 above the shelf but lets 3 and 4 fall.
 What are the missing numbers? Try the same thing with different numbers.
- With Gravity off, try to make 5 all-at-once. Can you do the same for 7?
- Teacher places six fingers successively on the screen without letting children see (but they will hear the numbers being said aloud). Show the screen to the children and ask how many yellow circles there are.
- Place all the even numbers on the Shelf, letting the odd numbers fall away. Continue doing this until you develop a rhythm for doing it quickly. Now **Reset** and make the odd numbers. How was this similar to the even numbers, and how different?
- Can you make the voice count by 5s? By 10s?

(From: http://touchcounts.ca/numbers.html)

71. Elections

There are two parties in an election, Red and Blue. There are only five people voting, and they are numbered 1, 2, 3, 4 and 5. What's interesting is that the person's number counts as the number of votes that they cast. When the votes were counted:

- more people had voted for Red
- Red had the more votes than Blue
- if any one person had changed their vote, then Blue would have won.

What are all the possible ballot counts for this situation?

Extensions: What about 6 people, 7 people, and so on? What if there were three parties?

(From: http://mathpickle.com/project/virtuous-democracy-addition-logic/)

72. **Tic-Tac-Toe Products Number of Players:** 2 **Material:**

- two markers for the bottom row of factors (for example, paper clips)
- two sets of different markers for each player to cover each product (for example, pennies and nickels)
- Tic-Tac-Toe Products Game Sheet (http://bit.ly/2rzNMSg)

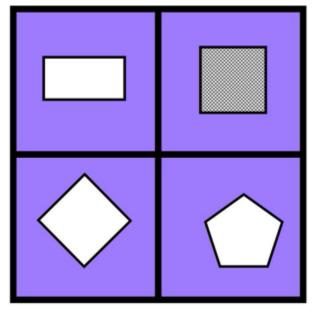
Task Instruction

- Player X and Player O select one factor, 1 9 at the bottom of the page and place one of the markers on that factor.
- Player X may move only one of the two markers to a new factor. Player X then places a marker on the grid covering the product of the two factors.
- Player O may move only one marker to make a new product and place their marker on the grid. The markers can both be placed on the same factor. For example: $6 \times 6 = 36$.
- Players alternate moving one factor marker at a time and continue placing their markers on the grid until a player has marked four products in a row. After the game, players should discuss their strategies.

(From youcubed.org: http://bit.ly/2sEDgYX)

G1. Which one doesn't belong?

Display this image for the class and ask "Which one doesn't belong?"



This task promotes students using descriptive language in their reasoning, and class discussion around multiple answers.

(From: www.wodb.ca)

73. **Sets of Numbers**

How many different sets of numbers with at least four members can you find in the numbers in this box?

For example, one set could be multiples of 4 {8,36...}, another could be odd numbers {3,13...}.

Extensions: What about "at least 5 members?"

(From: https://nrich.maths.org/1175)

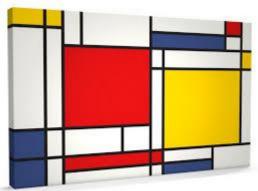
74. The Shoe Sale

You decide to take advantage of a buy 2 pair get 1 pair of equal or lesser value for free sale at the local shoe store. The problem is that you only want to get two pairs of shoes. So, you bring your best friend with you to the store. After much deliberation, you settle on two pairs of shoes – a sporty red pair for \$20 and a dressy black pair for \$55. Your friend finds a practical cross trainer for \$35. When you proceed to the check out desk the cashier tells you that your bill is \$90 plus tax (the \$20 pair are for free). How much should each of you pay? Justify your decision.

(From: http://www.peterliljedahl.com/teachers/numeracy-tasks)

H1. Mondrian Art

Piet Mondrian was one of the greatest painters and the leading abstract artist of the 20th century.



Show the class this example of Mondrian's art and ask, "What do you notice?"

Ask students to create their own Mondrian art using this application on their iPads: http://bit.ly/2r8tLBd

When complete, students can reflect on how their art is similar and different to the Mondrian example.

(From: https://www.sfu.ca/geometry4yl.html)

75. **Cycling Shop**

Imagine you work at a cycling shop building unicycles, bicycles, and tricycles for customers. One day, you receive a shipment of 8 wheels. Presuming that each cycle uses the same type and size of wheel, what are all the combinations of cycles you can make using all 8 wheels?

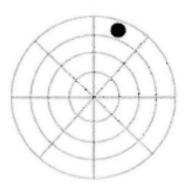
Extension:

What if you had more wheels?

(From: Mike Flynn https://tinyurl.com/y8gdvvrh and Alicia Burdess https://tinyurl.com/yan2t5jn)

76. **Hockey Puck Board Game**

Ferin and Ian play a game by alternately moving a hockey puck on a board with n concentric circles divided into r regions. For example, in the diagram below, we have n=4 and r=8. The game starts with the puck already on the board, as shown. A player may move either clockwise one position or one position towards the centre, but cannot move to a position that has been previously occupied. The last person who is able to move wins the game. Ferin moves first. Can Ferrin win? Explore.



(From: Richard Hoshino)

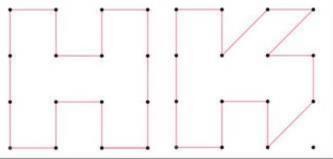
77. An eccentric woman checks into a hotel. She has booked three rooms all adjacent and connected to each other. As she is finishing the check in process she says to the manager, "If you need me I will always be in the room next to the room I was in the day before". The manager thinks nothing of this, but after the woman has gone to her room he realizes that her credit card has been declined. So, he has to find her. But he is very busy and only has time to knock on one room per day. What is the sequence of doors he has to knock on to ensure that he finds her? What if there were four rooms? Five? And so on. What if there was 17 rooms and she was leaving on the 30th day – can you find her?

78. • Start with any whole number

- If the number is even, divide it by 2 (or halve it)
- If the number is odd, multiply it by 3 and add 1
- Continue generating numbers until your sequence ends
- Choose another number and create the sequence. What do you think will happen?

<u>Paul Erdős</u> said about the Collatz conjecture: "Mathematics may not be ready for such problems." He also offered \$500 for its solution. <u>[9] Jeffrey Lagarias</u> in 2010 claimed that based only on known information about this problem, "this is an extraordinarily difficult problem, completely out of reach of present day mathematics." <u>[10]</u>

- 79. Given an 11 by 13 rectangle drawn on a grid, what is the smallest number of squares that can fill this rectangle?
- 80. A polygon is a shape where each side is made from a straight line. The H polygon below has 12 sides and the K has 13 sides. (Note: Each side of the polygon must join two dots. Lines cannot overlap. The shape must have no gaps in its outline, and each dot can be visited at most once.) What is the "most-sided" polygon that can be drawn on a 4 by 4 grid?



81. Lewis Carroll posed the following problem: If 6 cats can kill 6 rats in 6 minutes, ho many cats are required to kill 100 rats in 50 minutes?